$$=\left(\frac{\pi}{16}-\tfrac{1}{8}\right)/\frac{\pi}{4}=\tfrac{1}{2}\left(\tfrac{1}{2}-\frac{1}{\pi}\right).$$

(II) In this case the *superior* limit of ϕ in the numerator of C_1 is the value of ϕ derived from the equation $\sin\phi\cos^3\phi=\frac{1}{16}\pi$; and the *inferior* limit of the same variable is zero.

The required chance C_2 can, therefore, be found approximately; but is not of sufficient interest to warrant the labor required to find it.

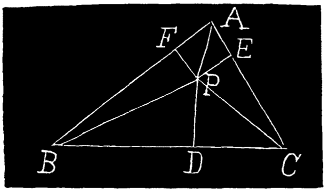
MISCELLANEOUS.

147. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

If P be a point within the scalene triangle, such that $\angle PAB=\angle PBC=\angle PCA=\psi$, then $\cot\psi=\cot A+\cot B+\cot C$.........(1), and $\text{cosec}^2\psi=\text{cosec}^2A+\text{cosec}^2B+\text{cosec}^2C$.........(2).

I. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Let $\angle PAB=\angle PCA=\angle PBC=\psi$. Then $\angle APB=\pi-(\psi+B-\psi)=\pi-B$. Draw PD, PE, PF perpendicular to BC, CA, AB, respectively.



$$PD=PB\sin\psi=\frac{AB\sin PAB}{\sin APB}.\sin\psi=\frac{c\sin^2\psi}{\sin B}$$

$$=\frac{2Rc}{b}\sin^2\psi.\quad \text{So } PE=2R\frac{a}{b}\sin^2\psi,$$

$$PF=2R\frac{b}{a}\sin^2\psi.$$

$$\frac{\sin(A-\psi)}{\sin\psi}=\frac{PE}{PF}=\frac{a^2}{bc}=\frac{\sin A\sin(B+C)}{\sin B\sin C}.$$

$\therefore \cot\psi-\cot A=\cot B+\cot C$, or $\cot\psi=\Sigma\cot A$.

Also $\cot^2\psi=\Sigma\cot^2A+2\Sigma\cot B\cot C$, $\text{cosec}^2\psi-1=\Sigma\text{cosec}^2A-3+2$;

i. e., $\text{cosec}^2\psi=\Sigma\text{cosec}^2A$.

II. Solution by the PROPOSER.

Let $PA=m$, $PB=n$, $PC=p$. $\sin(\pi-B):\sin(B-\phi)=c:m$.

$\therefore \cot\phi-\cot B=(m/c\sin\phi)=2\cot B$ ....(1).

Also, $\cot\phi-\cot C=(p/a\sin\phi)=2\cot C$..........(2);

and $\cot\phi-\cot A=(n/b\sin\phi)=2\cot A$..........(3).

Adding, and dividing by (3), we have $\cot\phi=\cot A+\cot B+\cot C$..........($A$).

Squaring (A), and transforming into cosecants, we have

$$\text{cosec}^2\psi=\text{cosec}^2A+\text{cosec}^2B+\text{cosec}^2C.$$

Also solved by M. E. Graber, J. Scheffer, and A. H. Holmes.